

FRICTION

Property of oppose b/w relative motion of +1/0 surface.

Type of friction

- * Internal Friction → Friction b/w two surface of same mat. ⇒ viscosity.
- * External Friction → b/w surface of two different solid.
- |a| → Static friction → * When body is at rest, friction force b/w two layer of solid is self adjusting friction & its max value is called limiting friction.
- * $F_L = (F_s)_{max}$
- |b| → Dynamic/Kinetic friction → When body move fr. b/w two surface of solid.

Limiting friction depend on → * It is \propto Normal RKN .
 * It depend on nature of surface.
 * Independent from surface or contact area.

Max value of static friction → $F_L \propto N$

$$F_L = (F_s)_{max} = \mu_s N$$

μ_s = static friction coefficient.

Kinematic friction →

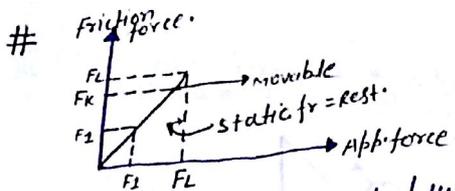
$$F_k \propto N \quad F_k = \mu_k N$$

$$F_L > F_k \Rightarrow \mu_s > \mu_k$$

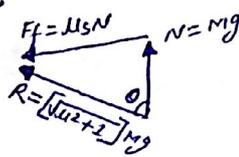
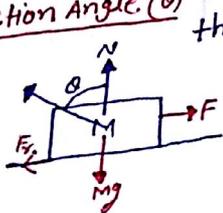
NOTE →

- * $(F_{ex})_{app} < F_L = (F_s)_{max} \Rightarrow$ Body does not move $\Rightarrow F \cdot F = (F_{ex})_{app}$.
- * $(F_{ex})_{app} = F_L = (F_s)_{max} \Rightarrow F \cdot F = F_L = (F_s)_{max}$
- * $(F_{ex})_{app} > F_L \Rightarrow$ body moves on surface $\Rightarrow F \cdot F = F_k$

* order of friction → $\mu_s > \mu_k > \mu_r$



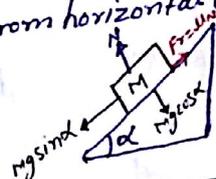
Friction Angle (θ) → Angle b/w resultant of Normal RKN & friction force normal to the surface.



$$\tan \theta = \frac{F_f}{N} = \frac{\mu_s N}{N} = \mu_s$$

$$\theta = \tan^{-1}(\mu_s)$$

Angle of Repose → Angle of plane surface from horizontal. (If body just slide on surface).



* Just slide on surface.

$$Mg \sin \alpha = F_f = \mu_s N$$

$$Mg \sin \alpha = \mu_s (Mg \cos \alpha)$$

$$\tan \alpha = \mu_s$$

Angle of Repose = Angle of friction.

NOTE

- |a| → $\alpha = 0 \Rightarrow$ Just slide on surface $\Rightarrow F \cdot F \Rightarrow \mu_s Mg \cos \alpha$
- |b| → $\alpha < 0 \Rightarrow$ Body present at rest $\Rightarrow F \cdot F \Rightarrow Mg \sin \alpha$.
- |c| → $\alpha > 0 \Rightarrow$ Body moving downward $\Rightarrow F \cdot F \Rightarrow F_f = \mu_k = \mu_k Mg \cos \alpha$

$$a = g \sin \alpha - \mu_k \cos \alpha$$

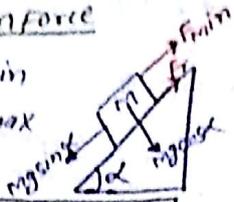
$$Mg \sin \alpha > \mu_s Mg \cos \alpha$$

Minimum & Maximum force required to hold the object in a incline plane (Force applied parallel to the incline surface).

* Minimum force

$$Mg \sin \alpha = F + F_{min}$$

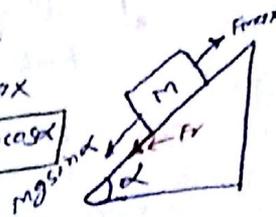
$$F_{min} = Mg \sin \alpha - (F)_{max}$$



* Maximum force

$$F_{max} = Mg \sin \alpha + (F)_{max}$$

$$F_{max} = Mg \sin \alpha + \mu_s Mg \cos \alpha$$



$$F_{min} = Mg \sin \alpha - \mu_s Mg \cos \alpha$$

Body is drop from top point of inclined plane of angle of inclination is 'theta'. If time taken by particle to reached at bottom resp. t1 & t2 when surface is smooth & rough & Relation t1 & t2. t2 = n times of t1. Friction coefficient of rough surface.

$$t_1 = \sqrt{\frac{2L}{g \sin \theta}}$$



$$t_2 = \sqrt{\frac{2L}{g \sin \theta - \mu \cos \theta}}$$



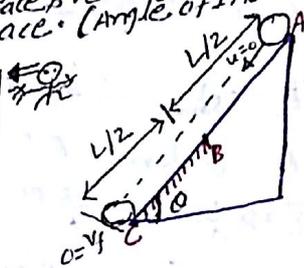
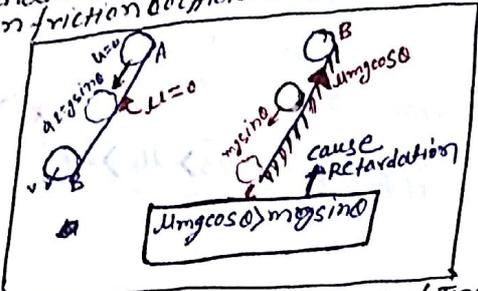
$$\mu = \tan \theta \left(1 - \frac{1}{n^2} \right)$$

Exemplar:

$$\mu = \left(1 - \frac{1}{n^2} \right)$$

First half length of incline plane is smooth & remaining half is rough. If particle is drop from top point of incline plane along the incline surface & velocity of particle at bottom point is zero. Then friction coefficient of rough surface. (Angle of inclination 'theta')

$$\mu = 2 \tan \theta$$



Particle is projected upward along the surface of inclined plane, friction coefficient is $\frac{1}{\sqrt{2}}$ & angle of inclination is 45° . Calculate retardation of particle along the incline plane.

$$F = Mg \sin \theta + \mu Mg \cos \theta$$

$$a_R = g \sin \theta + \mu g \cos \theta$$

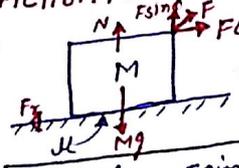
$$= g \sin 45^\circ + \left(\frac{1}{\sqrt{2}} \right) g \cos 45^\circ$$

$$R = \frac{g}{\sqrt{2}} + \frac{g}{2} = \frac{g}{2} (\sqrt{2} + 1)$$



Friction Force & Accel of Body.

|A| ->



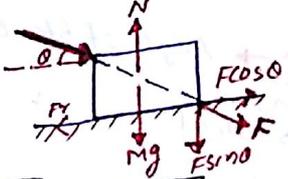
$$N = Mg - F \sin \theta$$

$$F_f = \mu N$$

$$F_f = \mu (Mg - F \sin \theta)$$

$$a = \frac{F_{net}}{M_{net}} = \frac{F \cos \theta - \mu (Mg - F \sin \theta)}{M}$$

|B| ->



$$N = Mg + F \sin \theta$$

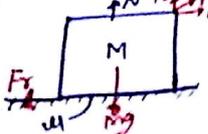
$$F \sin \theta > F_f$$

$$F_f = \mu N = \mu (Mg + F \sin \theta)$$

$$a = \frac{F_{net}}{M_{net}} = \frac{F \cos \theta - \mu (Mg + F \sin \theta)}{M}$$

NOTE -> pull is easier than push.

Minimum Force Required to pull the object.

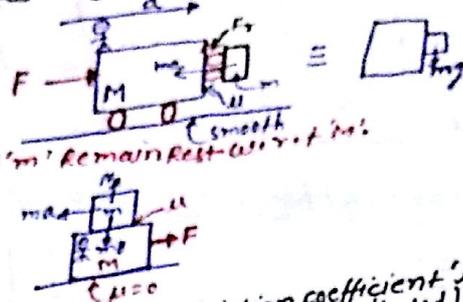


$$\mu_s = \tan \theta$$

$$\theta = \tan^{-1} (\mu_s)$$

$$F_{min} = \frac{\mu_s Mg}{\sqrt{1 + \mu_s^2}}$$

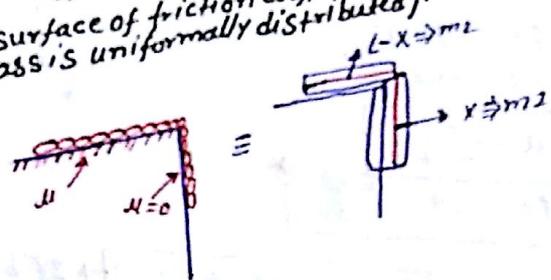
Magnitude of minimum force required if mass 'm' is at rest w.r.t. mass 'M'.
 $N = ma$
 $a \geq \mu g$
 $a_{min} = \mu g$
 $F_{min} = (M+m)a_{min} = \frac{(M+m)g}{\mu}$



Max value of force applied on mass 'm'. So that 'm' remains rest w.r.t. 'M'.
 $F \geq ma$
 $a \leq \mu g$
 $a_{min} = \mu g$
 $F_{max} = (M+m)a = \mu(M+m)g$
 $F > \mu(M+m)g$

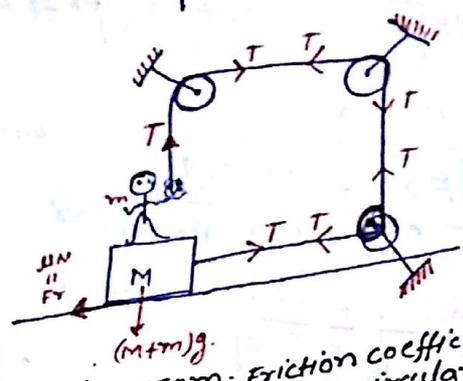


Chain of length 'L' placed on rough horizontal surface of friction coefficient 'u' than maximum length of its hanging part. If it remains in rest (mass is uniformly distributed).
 $L = M$
 $l = m$
 $x = \frac{m}{L}x = m/2$
 $L-x = \frac{M}{L}(L-x) = m/2$
 $x = \frac{\mu L}{\mu + 1}$



NCERT # Force applied by person to prevent slipping.

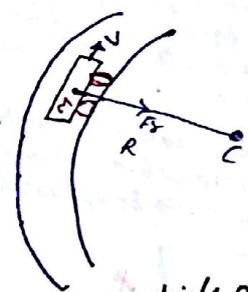
$F \geq T$
 $T \leq \frac{\mu(M+m)g}{\mu + 1}$
 $T_{max} = F_{max} = \frac{\mu(M+m)g}{\mu + 1}$



*** # vehicle of mass 800 kg moves on circular path of radius 10m. Friction coefficient of surface is 0.5. calculate maximum safe velocity of vehicle on circular path.

$F_r \geq \frac{mv^2}{R}$
 $v \leq \sqrt{\mu Rg}$
 $v_{max} = \sqrt{\mu Rg}$

NOTE -> * In a circular path necessary centripetal force is provided from fr. force.
 * Magnitude of fr. force > centrifugal force vehicle safely turn on circular path.

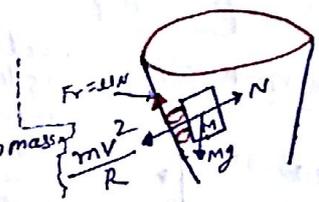


$v_{max} = \sqrt{(0.5)^2 + (10)(20)}$
 $= \sqrt{50} \text{ m/sec}$

*** # Radius of death well is 10m & minimum safe velocity of vehicle on death well is $5\sqrt{5} \text{ m/sec}$.
 than value of friction coefficient.

$F_r \geq Mg$
 $\mu N \geq Mg$
 $v \geq \sqrt{\frac{Rg}{\mu}}$
 $v_{min} = \sqrt{\frac{Rg}{\mu}}$
 $\mu_{min} = \frac{Rg}{v_{min}^2}$

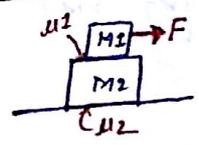
NOTE -> * Max safe velocity is independent from of vehicle/man mass.
 $v \propto m^0$



$= \frac{10 \times 10}{(5\sqrt{5})^2} = \frac{100}{125} = \frac{4}{5} = 0.8$

Friction in two block system
 Case - I -> Force is applied on upper block

[A] -> $\mu = 0$
 $a = \frac{F}{M_1 + M_2}$



* If both block move with same acceleration.
 $F < f_1$
 $F \leq \mu(M_1 + M_2)g$

*** $F > f_1$ -> Both block move with diff. Acceleration.

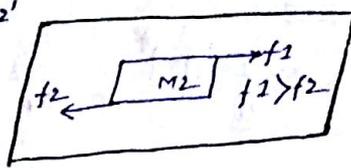
$F - f_1 = M_1 a$ (1)
 $f_1 = M_2 a$ (2)

|B| → $\mu_2 \neq 0$

sliding condn for 'M2'

$$f_1 \geq \mu_2 (M_1 + M_2)g$$

$$\mu_1 M_1 g \geq \mu_2 (M_1 + M_2)g$$



Case-II → Force applied on lower block

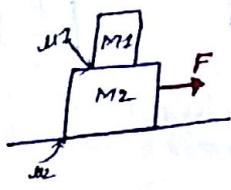
[A] → $\mu_2 = 0$

$$f_1 = M_1 a \text{ (i)}$$

$$F - f_1 = M_2 a \text{ (ii)}$$

$$a = \frac{F}{M_1 + M_2}$$

$$f_1 = M_1 a = \frac{M_1 F}{M_1 + M_2}$$



$$f_1 \geq M_1 a$$

$$\mu_1 M_1 g \geq \frac{M_1 F}{M_1 + M_2}$$

$$F \leq \mu_1 (M_1 + M_2)g$$

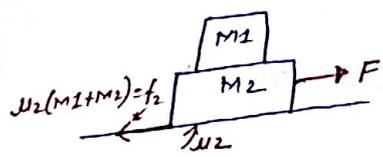
→ If both block move with same acceleration.

$F > \mu_1 (M_1 + M_2)g$ → If both block move with different acceleration.

[B] → $\mu_2 \neq 0$

$$F > \mu_2 (M_1 + M_2)g$$

→ sliding condn for 'M2'

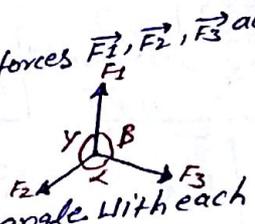


Equilibrium of Body.

$$\sum \vec{F} = 0, \sum \vec{\tau} = 0$$

Lamm's theorem → When 3-concurrent forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ act on body in eqn. there

$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

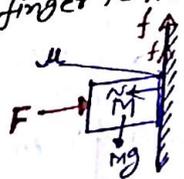


NOTE → * If 'n' coplaner forces of equal magnitude act on body at equal angle with each other than net force it will be zero. hence body is in eqn.
* If 'n' force act on a body & these forces form closed polygonal then resultant force will be zero.

A block of mass 'M' is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction b/w block & wall is ' μ ' & Acceleration due to gravity is 'g'. than minimum force required to be applied by the finger to hold block against the wall.

$$\mu F = Mg$$

$$F = \frac{Mg}{\mu}$$



Rolling friction → * Oppose of Rolling Motion.
* Magnitude is less than from kinetic friction.

$$F_R = \frac{\mu R N a}{r}$$

Normal Reaction. Radius.

2016 JEEB NOTE → In case of pure Rolling motion, Work done by fr. force will be zero, so in this case friction is non-dissipated. (not loss).
* When body roll on horizontal plane the velocity of contact point remain zero.

NCERT Exemplar #

$$m = \frac{F}{a}$$

Inertial mass $m_i = \frac{F}{a}$

Gravitation mass $m_g = \frac{F}{g}$

$$\frac{m_i}{m_g} = 1:1$$

*** NOTE → * Every body have Inertia (ie mass) but its weight (mg) can be zero.
* In a tunnel through the centre of the earth, the object moves only due to Inertia at the centre while its weight becomes zero.

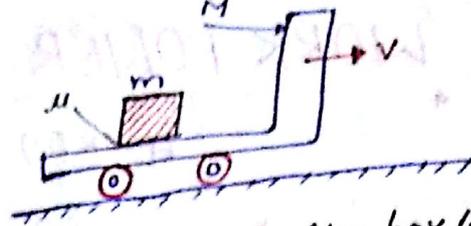
* v_{max} so that block remain stationary with truck.

$$v_{max} \geq ma$$

$$\mu_1 mg \geq ma$$

$$\mu \geq a$$

$$\mu_s \geq a/g$$



* If truck accelerate & box start moving backward than time after box leave the truck

$$t = \sqrt{\frac{2L+l}{\mu g}}$$

[L = length of truck
 l = length of box]